$$
\begin{array}{lll}
t & ::= & \text{型} \\
 & \texttt{int} & \text{整数型} \\
 & \texttt{bool} & \text{真偽値型} \\
 & t \to t & \text{関数型} \\
 & \alpha & \text{型変数}
\end{array}
$$

図 1: 型の定義

$$\frac{n\ \text{は整数値}}{\Gamma \vdash n : \texttt{int}, \emptyset} \qquad \frac{b\ \text{は真偽値}}{\Gamma \vdash b : \texttt{bool}, \emptyset} \qquad \frac{\Gamma(x) = t}{\Gamma \vdash x : t, \emptyset}$$

$$\frac{\Gamma \vdash e_1 : t_1, C_1 \quad \Gamma \vdash e_2 : t_2, C_2 \quad (\mathit{aop}\ \text{は整数演算})}{\Gamma \vdash e_1\ \mathit{aop}\ e_2 : \texttt{int}, \{t_1 = \texttt{int}\} \cup \{t_2 = \texttt{int}\} \cup C_1 \cup C_2}$$

$$\frac{\Gamma \vdash e_1 : t_1, C_1 \quad \Gamma \vdash e_2 : t_2, C_2 \quad (\mathit{bop}\ \text{は論理演算})}{\Gamma \vdash e_1\ \mathit{bop}\ e_2 : \texttt{bool}, \{t_1 = \texttt{bool}\} \cup \{t_2 = \texttt{bool}\} \cup C_1 \cup C_2}$$

$$\frac{\Gamma \vdash e_1 : t_1, C_1 \quad \Gamma \vdash e_2 : t_2, C_2}{\Gamma \vdash e_1 = e_2 : \texttt{bool}, \{t_1 = t_2\} \cup C_1 \cup C_2}$$

$$\frac{\Gamma \vdash e_1 : t_1, C_1 \quad \Gamma \vdash e_2 : t_2, C_2 \quad \Gamma \vdash e_3 : t_3, C_3}{\Gamma \vdash \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 : t_2, \{t_1 = \texttt{bool}\} \cup \{t_2 = t_3\} \cup C_1 \cup C_2 \cup C_3}$$

$$\frac{\Gamma \vdash e_1 : t_1, C_1 \quad \Gamma, x : t_1 \vdash e_2 : t_2, C_2}{\Gamma \vdash \texttt{let}\ x = e_1\ \texttt{in}\ e_2 : t_2, C_1 \cup C_2} \qquad \frac{\begin{array}{c}\Gamma, x : \alpha \vdash e_1 : t_1, C_1 \quad \Gamma, x : t_1 \vdash e_2 : t_2, C_2 \\ (\alpha\ \text{は新しい型変数})\end{array}}{\Gamma \vdash \texttt{let rec}\ x = e_1\ \texttt{in}\ e_2 : t_2, \{\alpha = t_1\} \cup C_1 \cup C_2}$$

$$\frac{\begin{array}{c}\Gamma \vdash e_1 : t_1, C_1 \quad \Gamma \vdash e_2 : t_2, C_2 \\ (\alpha\ \text{は新しい型変数})\end{array}}{\Gamma \vdash e_1\ e_2 : \alpha, \{t_1 = t_2 \to \alpha\} \cup C_1 \cup C_2} \qquad \frac{\Gamma, x : \alpha \vdash e : t, C}{\Gamma \vdash \texttt{fun}\ x\ \texttt{=}\ e : \alpha \to t, C}$$

(Γ は変数から型への写像を表す)

図 2: 型推論規則 (多相型なし)

$$\frac{\Gamma(x) = \forall\Delta.t \quad t' = [\Delta'/\Delta]t \\ (\Delta' \text{ は新しい型変数の集合})}{\Gamma \vdash x : t', \emptyset}$$

$$\frac{\begin{array}{c}\Gamma \vdash e_1 : t_1, C_1 \quad \Gamma', x : t_s \vdash e_2 : t_2, C_2 \\ \text{ただし} \\ S = \mathit{Unify}(C_1) \\ t'_1 = S(t_1) \quad \Gamma' = S(\Gamma) \\ \Delta = FV(t'_1) \backslash FV(\Gamma') \quad t_s = \forall\Delta.t'_1\end{array}}{\Gamma \vdash \texttt{let } x = e_1 \texttt{ in } e_2 : t_2, C_1 \cup C_2}$$

$$\frac{\begin{array}{c}\Gamma, x : \forall\emptyset.\alpha \vdash e_1 : t_1, C_1 \quad \Gamma', x : t_s \vdash e_2 : t_2, C_2 \\ \text{ただし} \\ S = \mathit{Unify}(C_1 \cup \{\alpha = t_1\}) \\ t'_1 = S(t_1) \quad \Gamma' = S(\Gamma) \\ \Delta = FV(t'_1) \backslash FV(\Gamma') \quad t_s = \forall\Delta.t'_1 \\ (\alpha \text{ は新しい型変数})\end{array}}{\Gamma \vdash \texttt{let rec } x = e_1 \texttt{ in } e_2 : t_2, \{\alpha = t_1\} \cup C_1 \cup C_2}$$

...その他の規則は同じ

Γ は変数から型スキーマ ($\forall\Delta.t$) への写像、Δ は型変数の集合、
$[t'/\alpha]t$ は型 t 中の自由な型変数 α を型 t' で置き換えたもの
($[t_1, \ldots, t_n/\Delta]t$ はこれを型変数の集合の置き換えに拡張したもの)、
Unify は制約の解を返す関数、$FV(t)$ は型 t 中の自由な型変数の集合を表す

図 3: 型推論規則 (多相型あり)